Chapter 9

Right Triangles and Trigonometry

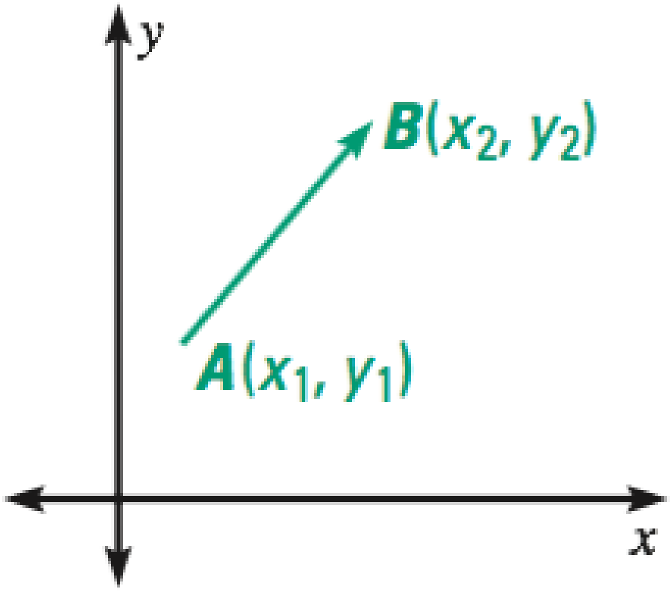
Section 7
Vectors

GOAL 1: Finding the Magnitude of a Vector

As defined in Lesson 7.4, a *vector* is a quantity that has both magnitude and direction. In this lesson, you will learn how to find the *magnitude of a vector* and the *direction of a vector*. You will also learn how to add vectors.

The **magnitude of a vector** $\overrightarrow{AB}$ is the distance from the initial point A to the terminal point B, and is written $|\overrightarrow{AB}|$. If a vector is drawn in a coordinate plane, you can use the Distance Formula to find its magnitude.

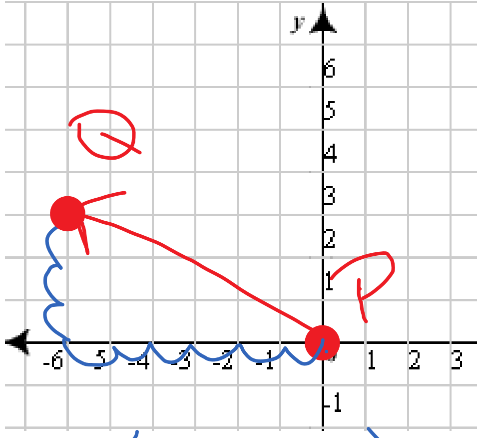
$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1: Finding the Magnitude of a Vector

Points P and Q are the initial and terminal point of the vector PQ. Draw PQ in a coordinate plane. Write the component form of the vector and find its magnitude.

a) P(0, 0), Q(-6, 3)



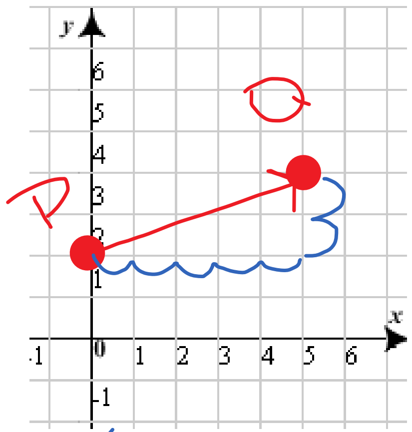
$\langle -6, 3 \rangle$

$\sqrt{(-6)^2 + 3^2}$

$\sqrt{45}$

6.7

b) P(0, 2), Q(5, 4)



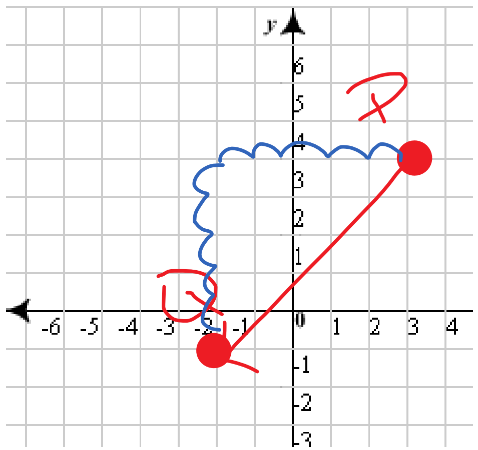
$\langle 5, 2 \rangle$

$\sqrt{5^2 + 2^2}$

$\sqrt{29}$

5.4

c) P(3, 4), Q(-2, -1)



$\langle -5, -5 \rangle$

$\sqrt{(-5)^2 + (-5)^2}$

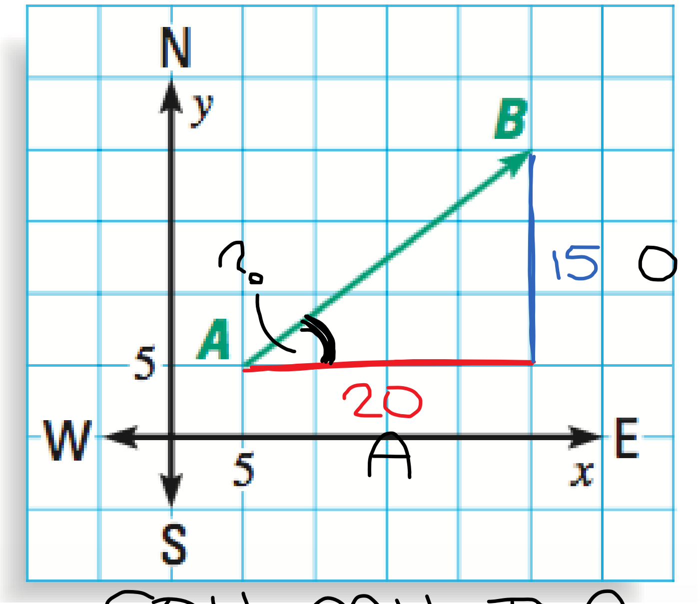
$\sqrt{50}$

7.1

The **direction of a vector** is determined by the angle it makes with a horizontal line. In real-life applications, the direction angle is described relative to the directions north, east, south, and west. In a coordinate plane, the x-axis represents an east-west line. The y-axis represents a north-south line.

Example 2: Describing the Direction of a Vector

The vector AB describes the velocity of a moving ship. The scale on each axis is in miles per hour.



SOH CAH TOA

a) Find the speed of the ship.

$$\sqrt{20^2+15^2} \rightarrow \sqrt{400+225}$$

$$\rightarrow \sqrt{625} \rightarrow 25$$

b) Find the direction it is traveling relative to east.

$$\tan A = \frac{15}{20}$$

$$A = \tan^{-1}\left(\frac{15}{20}\right)$$

$$36.9^\circ$$

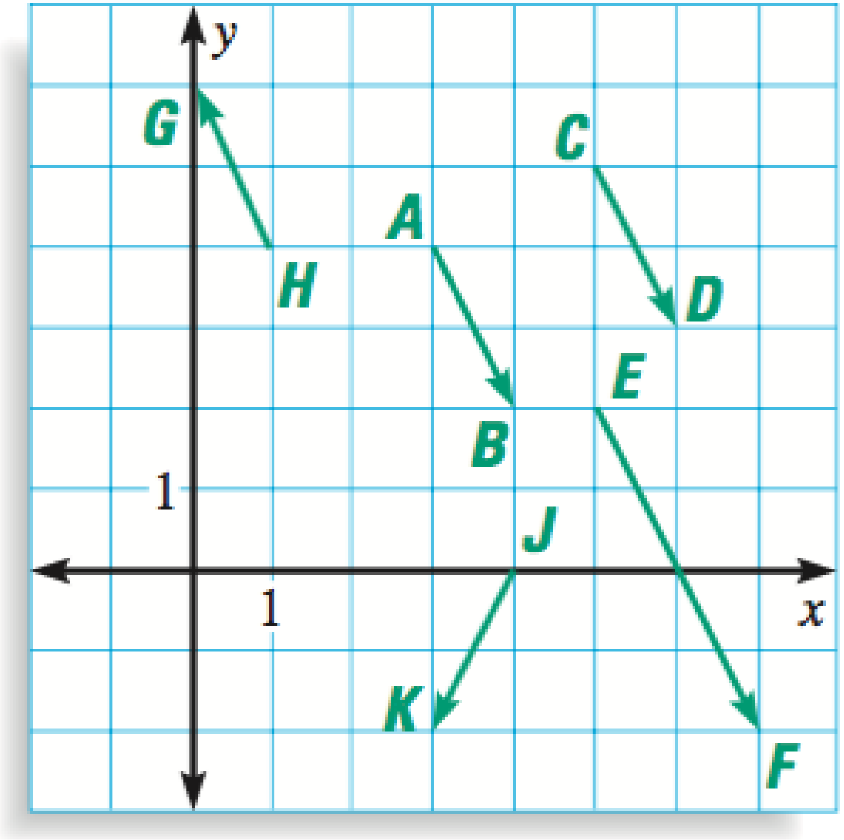
Two vectors are **equal** if they have the same magnitude and direction. They do *not* have to have the same initial and terminal points. Two vectors are **parallel** if they have the same or opposite directions.

Example 3: Identifying Equal and Parallel Vectors

In the diagram, these vectors have the same direction: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$.

These vectors are equal: $\overrightarrow{AB}$, $\overrightarrow{CD}$.

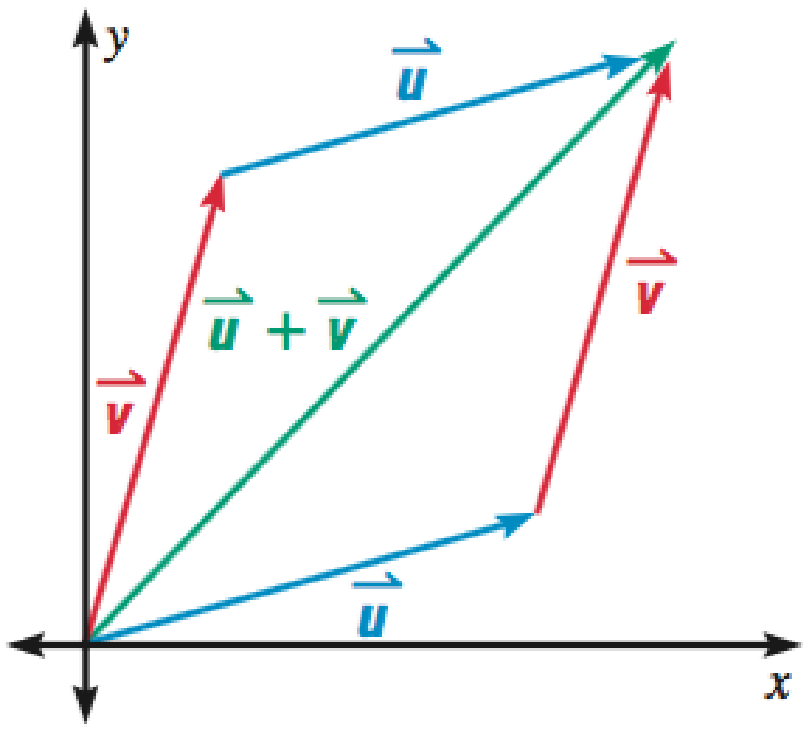
These vectors are parallel: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{HG}$.



GOAL 2: Adding Vectors

Two vectors can be added to form a new vector. To add $\vec{u}$ and $\vec{v}$ geometrically, place the initial point of $\vec{v}$ on the terminal point of $\vec{u}$, (or place the initial point of $\vec{u}$ on the terminal point of $\vec{v}$). The sum is the vector that joins the initial point of the first vector and the terminal point of the second vector.

This method of adding vectors is often called the *parallelogram rule* because the sum vector is the diagonal of a parallelogram. You can also add vectors algebraically.



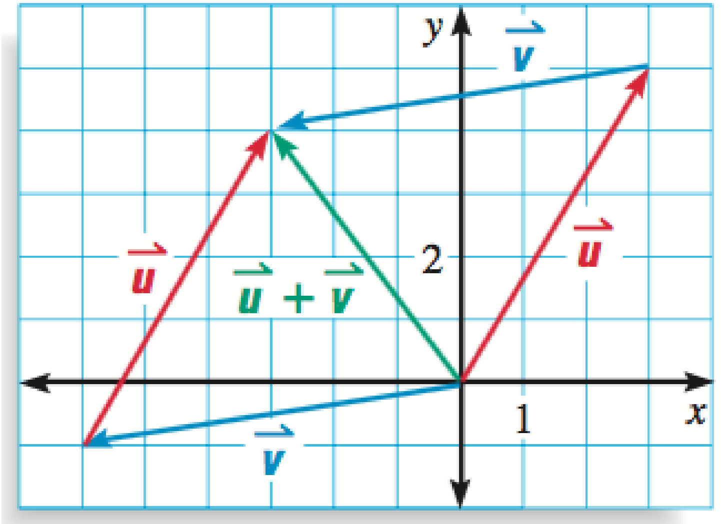
ADDING VECTORS

SUM OF TWO VECTORS

The **sum** of $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$ is $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$.

Example 4: Finding the Sum of Two Vectors

Let $\vec{u} = \langle 3, 5 \rangle$ and $\vec{v} = \langle -6, -1 \rangle$. To find the sum vector $\vec{u} + \vec{v}$, add the horizontal components and add the vertical components of $\vec{u}$ and $\vec{v}$.



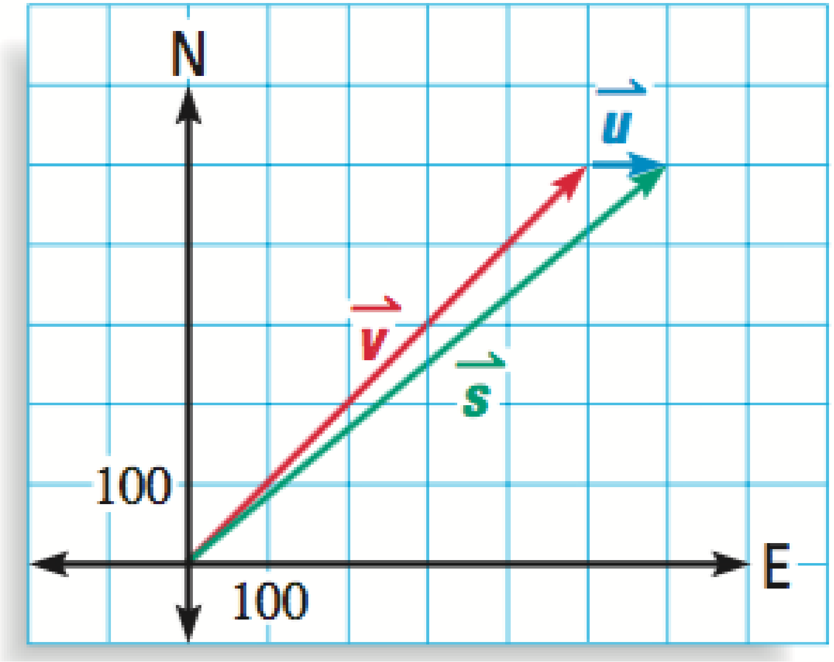
$$\langle 3 + {}^{-}6, 5 + {}^{-}1 \rangle$$

$$\langle -3, 4 \rangle$$

Example 5: Velocity of a Jet

AVIATION A jet is flying northeast at about 707 miles per hour. Its velocity is represented by the vector $\vec{v} = \langle 500, 500 \rangle$.

The jet encounters a wind blowing from the west at 100 miles per hour. The wind velocity is represented by $\vec{u} = \langle 100, 0 \rangle$. The jet's new velocity vector $\vec{s}$ is the sum of its original velocity vector and the wind's velocity vector.



$u + v \rightarrow \langle 100+500, 0+500 \rangle$

$\langle 600, 500 \rangle$

$\sqrt{600^2 + 500^2}$

$\sqrt{360000 + 250000}$

$\sqrt{610000} \rightarrow 781$

EXIT SLIP